ALGORITHMS FOR SOLVING ASSIGNMENT PROBLEM

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Abstract:

We propose another calculation for the old style assignment problem. The calculation takes after here and there the Hungarian strategy however varies considerably in different regards. The normal computational multifaceted nature of a proficient execution of the calculation is by all accounts impressively superior to the one of the Hungarian strategy. In countless arbitrarily produced problems the calculation has reliably outflanked a proficiently coded variant of the Hungarian strategy by an expansive edge. The assignment problem was among the principal direct programming problems to be contemplated broadly. It emerges frequently by and by and it is a principal problem in system stream hypothesis since various different problems, for example, the most brief way, weighted coordinating, transportation and least cost stream problems, can be diminished to it. It is trademark in this regard the main specific strategy for the assignment problem, to be specific Kuhn's Hungarian technique, was in this manner stretched out for arrangement of significantly more broad system stream problems. Besides, a portion of its primary thoughts were instrumental in the advancement of progressively broad strategies, for example, the out-of-kilter and non-bipartite coordinating techniques. This recommends the assignment problem isn't just significant in it, but on the other hand is appropriate for advancement of new computational thoughts in system stream hypothesis. It is hence that we confine thoughtfulness regarding the assignment problem despite the fact that the thoughts of this paper have expansions to progressively broad problems.

Keywords: *Assignment Problems, Network Flows, Hungarian Method, Computational Complexity.*

INTRODUCTION

The purpose for this paper is to propose another technique for solving the assignment problem. We appear in Section 3 that the most pessimistic scenario unpredictability of the unadulterated type of the strategy for full thick, all number, N × N problems with the components of the assignment network taking qualities in the interim [0, R] is O(N3) + O(RN2). It shows up, notwithstanding, that for enormous estimations of R (state R > 100) the strategy performs getting it done when it is joined with the Hungarian technique. One such blend is depicted in Section 4 and the most pessimistic scenario unpredictability of the subsequent technique is O(N3). Its normal unpredictability, in any case, is by all accounts considerably superior to the normal multifaceted nature of the Hungarian strategy. This is shown by methods for broad computational investigations with haphazardly produced problems. These analyses demonstrate that the new strategy reliably outflank s a proficiently executed rendition of the Hungarian technique by an expansive edge. Without a doubt, out of in excess of a thousand haphazardly produced problems tackled with N - > 20 we didn't locate a solitary problem where our strategy did not work quicker than the Hungarian technique. Besides, the factor of progress increments with N subsequently recommending a superior normal unpredictability. For huge problems with N being a few hundred our strategy can unite at least multiple times quicker than the Hungarian technique.

It appears that the at present most well known arrangement strategies for the assignment problem are particular types of the simplex technique [3-5] and adaptations of Kuhn's Hungarian strategy [6- - 8]. There has been some debate with respect to the general benefits of simplex codes and basic double (for example Hungarian) codes] 6, 9]. An ongoing computational examination [7] finds simplex and base double codes generally equivalent, while another investigation [8] reports that a basic double code dependent on crafted by Edmonds and Karp [10] outflanks simplex-like algorithms by a significant edge. The improvement of particular codes for the assignment problem utilizing refined programming procedures is a functioning examination territory so it is hard to evaluate the present cutting edge. The computational multifaceted nature of huge numbers of the current codes is obscure and in actuality a portion of these codes [3, 6] are exclusive. It is realized that the unpredictability of the Hungarian technique for full thick, all whole number, N × N assignment problems is O (N3) [I, p. 205]. There is no simplex sort strategy with unpredictability tantamount to O (N3) and there are no normal multifaceted nature gauges for either Hungarian strategies or simplex techniques to the degree of our insight.

Since we have been not able portray scientifically the normal multifaceted nature of our strategy we can't promise to completely comprehend the system of its quick union. On heuristic grounds, be that as it may, it gives the idea that the new technique owes its great execution basically to a marvel which we allude to as out evaluating. This is clarified all the more completely in the following area however fundamentally it alludes to a property of the technique whereby over the span of the calculation the costs of certain sinks are expanded by enormous augmentations - a lot bigger than in the Hungarian strategy. As a result these sinks are incidentally or for all time out valued by different sinks and are essentially determined out of the problem as in they don't get marked and filtered further- - in any event for generally prolonged stretch of time periods. Therefore the calculation requires less line operations since in actuality it manages a problem of lower measurement.

Mathematical Formulation of Assignment Problem

Every assignment problem has a framework related with it. For the most part the line contains the items or individuals we wish to dole out, and the segment involve the employments or undertakings we need them doled out to. Consider a problem of assignment of n assets to m exercises to limit the general expense or time so that every asset can connect with one and just one employment. The cost network ( ) is given as under:

Table 1. Approach of Assignment Problem

		Activity				
		A_1	A_2	......	A_n	Available
Resource	R_1	C_{11}	C_{12}	......	C_{1n}	1
	R_2	C_{21}	C_{22}	......	C_{2n}	1
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	R_n	C_{n1}	C_{n2}	......	C_{nn}	1
Required		1	1	......	1	

The cost matrix is same as that of a transportation problem except that availability at each of the resource and the requirement at each of the destinations is unity.

Let x_{ij} denote the assignment of i^{th} resource to j^{th}activity, such that

$$x_{ij} = \begin{cases} 1 \ ; & \text{if resource i is assigned to activity j} \\ 0 \ ; & \text{otherwise} \end{cases}$$

Then the mathematical formulation of the assignment problem is

Minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$

Subject to the constraints

$\sum_{i=1}^{n} x_{ij} = 1 \text{and} \sum_{j=1}^{n} x_{ij} = 1 \;:\; x_{ij} = 0 \text{ or } 1$

For all $i = 1,2, \dots \dots n$ and $j = 1,2, \dots \dots n.$

New Approach for Solving Assignment Problem

In this area we present another methodology for solving Assignment problem with the assistance of HA-technique and MOA-strategy however not the same as them. This new technique is simple strategy to take care of Assignment problem. Likewise a model is understood by this technique and the outcome is contrasted with HA-strategy and MOA-technique. Presently we consider the assignment lattice where cij is the expense of allotting ith employment to jth machine.

$$
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & \dots & n \\
\hline
1 & C_{11} & C_{12} & C_{13} & \dots\dots & C_{1n} \\
2 & C_{21} & C_{22} & C_{23} & \dots\dots & C_{2n} \\
. & . & . & . & \dots\dots & . \\
. & . & . & . & \dots\dots & . \\
. & . & . & . & \dots\dots & . \\
n & C_{n1} & C_{n2} & C_{n3} & \dots\dots & C_{nn} \\
\hline
\end{array}
$$

Proposed Method: Subtract Row and Add One Assignment Method

The proposed algorithm of proposed method is as follows:

Step 1: Find the smallest number (cost) of each row. Subtract this smallest number from every number in that row.

Step 2: Now add 1 to all element and we get at least one ones in each row. Then make assignment in terms of ones. If there are some rows and columns without assignment, then we cannot get the optimum solution. Then we go to the next step.

Step 3: Draw the minimum number of lines passing through all ones by using the following procedure:

i. Mark (√) rows that do not have assignments.

ii. Mark (√) columns that have crossed ones in that marked rows.

iii. Mark (√) rows that have assignments in marked columns.

iv. Repeat (b) and (c) till no more rows or columns can be marked.

v. Draw straight lines through all unmarked rows and marked columns

If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step 4: Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered numbers by this smallest number. Other numbers covered by lines remain unchanged. Then we get some new ones in row and column. Again make assignment in terms of ones.

Step 5: If we cannot get the optimal assignment in each row and column. Then repeat steps (3) and (4) successively till an optimum solution is obtained.

NUMERICAL ILLUSTRATIONS

In this section, we provide numerical examples to illustrate the proposed algorithm.

Example 1: The time in minutes required by the machines M1, M2, M3, and M4 to complete the Jobs J1, J2, J3 and J4 are given in the following table.

Machines	Jobs			
	J1	J2	J3	J4
M1	3	4	7	6
M2	4	6	8	9
M3	5	7	4	6
M4	7	9	10	3

Given CEM is a 4 x 4 matrix. Therefore, the process stops when given four jobs are assigned uniquely. We now calculate ROC as follows:

Machines	Jobs				Row Opportunity Cost
	J1	J2	J3	J4	
M1	3	4	7	6	(1)
M2	4	6	8	9	(2)
M3	5	7	4	6	(1)
M4	7	9	10	3	(4)

The most extreme Row Opportunity Cost relates to fourth line and the base expense in this column is C_{44}. In this way, we dole out machine M4 to work J_4 and erase fourth line and fourth segment. Rehashing the Step2 and Step3 in the calculation to the accompanying diminished lattice

Machines	Jobs			Row Opportunity Cost
	J1	J2	J3	
M1	3	4	7	(1)
M2	4	6	8	(2)
M3	5	7	4	(3)

The greatest Row Opportunity Cost in this emphasis compares to third line and the base expense in this line is C33. So dole out machine M3 to work J3 and erase the relating line and section. The lessen lattice is the most extreme Row Opportunity cost is 2 and the base cost cell in this line relates to (M2, J1).

Machines	Jobs		ROC
	J1	J2	
M1	3	4	(1)
M2	4	6	(2)

Finally, we are left with only one possibility of assigning M1 to J2. Hence, the optimal solution is achieved and is as follows

Machines	Jobs	Time
M1	J2	4
M2	J1	4
M3	J3	4
M4	J4	3

The optimal time taken to complete all the jobs is 4 + 4 + 4 + 3 =15.

Remark 2: One can check that the ideal arrangement got in Example 3.1 utilizing the proposed technique is same that of one gotten by Hungarian Method. Be that as it may, the proposed technique is straightforward and simple to actualize.

Notation: While applying the proposed technique to comprehend A.P we will demonstrate the diminished lines and segments by (- -) against them and doled out cells with square sections [ ]. The accompanying model shows the proposed strategy to tackle the lopsided Assignment Problem.

Example 3: Consider the following unbalanced A.P. with 4 x 3 CEM. The problem is to find the optimal assignment to the machines so that time taken to complete all the jobs is minimized.

Machines	Jobs		
	J1	J2	J3
M1	21	14	7
M2	15	10	5
M3	15	10	5
M4	12	8	4

First we balance the given AP by introducing a dummy column (Job J4) with zero cost values; thereafter we apply the proposed method as follows.

Machines	Jobs				Row Opportunity Cost		
	J1	J2	J3	J4	1st	2nd	3rd
M1	21	14	7	[0]	(7)	(--)	(--)
M2	15	10	[5]	0	(5)	(--)	(--)
M3	15	[10]	5	0	(5)	(5)	(--)
M4	[12]	8	4	0	(4)	(4)	(--)
Deleted Columns				(--)			
			(--)				
		(--)					

The optimal solution is as follows:

Machines	Jobs	Time
M1	J4	0
M2	J3	5
M3	J2	10
M4	J1	12

The optimal time taken to finish every one of the employments is 5 + 10 + 12 =27. One can without much of a stretch see that there is a tie in the open door cost of first cycle and we can choose either second column or third line as the base assignment cost in those lines is same. In the event that we select third line rather than second column in this emphasis, at that point an elective assignment plan is gotten and there will be no change in ideal time of finishing.

CONCLUSION:

In this paper, we displayed an Algorithm for Solving Assignment Problem. At first, we clarified the proposed calculation and demonstrated the productivity of it by numerical model. Consequently this paper presents an alternate methodology which is anything but difficult to take care of Assignment problem. Assignment problem is a significant problem in arithmetic and is likewise examine in genuine physical world. Likewise we think about the ideal arrangements among this new technique and two existing strategy. The proposed technique is a precise methodology, simple to apply for solving assignment problem. It is trademark in this regard the primary particular strategy for the assignment problem, was along these lines reached out for arrangement of substantially more broad system stream problems. Moreover, a portion of its principle thoughts were instrumental in the improvement of progressively broad techniques, for example, the out-of-kilter and non-bipartite coordinating strategies. This proposes the assignment problem isn't just significant in it, but on the other hand is appropriate for improvement of new computational thoughts in system stream hypothesis. It is consequently that we limit regard for the assignment problem despite the fact that the thoughts of this paper have augmentations to increasingly broad problems.

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