Cycle Related Mean Square Cordial Graphs

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Abstract – Let G = (V,E) be a graph with p vertices and q edges. A Mean Square Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label $(\lceil (f(u))^2 + (f(u))^2 \rceil)/2$ where ⌈ x ⌉ (ceilex) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by almost 1. The graph that admits a Mean Square Cordial Labeling is called Mean Square Cordial Graph. In this paper, we proved that cycle related graphs $(C_3 \otimes C_3)_m$, Crown $C_n \odot K_1$ (n-odd), Double Triangular Snake $C_2(P_n)$ (n-odd), Quadrilateral Snake Q(n) (n-odd) are Mean Square Cordial Graphs.

Keywords – Mean Square Cordial Graph, Mean Square Cordial Labeling.

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I.INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {u,v} of vertices in E is called edges or a line of G. In this paper, we proved that cycle related graphs $(C_3 \otimes C_3)_m$, Crown $C_n \odot K_1$ (n-odd), Double Triangular Snake $C_2(P_n)$ (n-odd), Quadrilateral Snake Q(n) (n-odd) are Mean Square Cordial graphs. For graph theory terminology, we follow [2].

II.PRELIMINARIES

Let G = (V,E) be a graph with p vertices and q edges. A Mean Square Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label $(\lceil (f(u))^2 + (f(u))^2 \rceil)/2$ where ⌈ x ⌉ (ceilex) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

The graph that admits a Mean Square Cordial Labeling is called Mean Square Cordial Graph. In this paper, we proved that cycle related graphs $(C_3 \otimes C_3)_m$, Crown $C_n \odot K_1$ (n-odd), Double Triangular Snake $C_2(P_n)$ (n-odd), Quadrilateral Snake Q(n) (n-odd) are Mean Square Cordial Graphs.

Definition: 2.1

$(C_n \otimes C_n)_m$ is a graph by joining C_n by an edge. Note that $(C_n \otimes C_n)_m$ has mn+m-1 edges and mn vertices.

Definition:2.2

The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th} copy of G. $C_n \Theta K_1$ is called a Crown.

Definition:2.3

Graph obtained from a path P_n,by joining each end vertices of an edge with two isolated vertex. It is denoted by $C_2(P_n)$.

Definition:2.4

A quadrilateral snake Q(n) is obtained from a path ($u_1, u_2, \ldots, u_n$) by joining u_i , u_{i+1} to new the vertices v_i and w_i respectively and joining the vertices u_i , v_i and u_i , w_i also joining the vertices u_{i+1} , v_i and u_{i+1} , w_i respectively. (i.e) every edge of the path is replaced by a cycle C_4.

III.MAIN RESULTS

Theorem: 3.1

$(C_3 \otimes C_3)_m$ (m-even) is Mean Square Cordial Graph.

Proof:

Let G be $(C_3 \otimes C_3)_m$

Let V(G) = { u_i , $1 \le i \le m$, u_{ij} : $1 \le i \le m$, $1 \le j \le 2$ }

Let E(G) = { [$(u_i u_{ij})$: $1 \le i \le m$, $1 \le j \le 2$] ∪ [$(u_{i1} u_{i2})$: $1 \le i \le m$] ∪ [$(u_{i2} u_{(i+1)1})$: $1 \le i \le m-1$]

Define f : V(G) → {0,1}

The vertex labeling are,

$$f(u_i) = \begin{cases} 0, 1 \le i \le \frac{m}{2} \\ 1, \frac{m+2}{2} \le i \le m \end{cases}$$

$$f(u_{ij}) = \begin{cases} 0, 1 \le i \le \frac{m}{2}, 1 \le j \le 2 \\ 1, \frac{m+2}{2} \le i \le m, 1 \le j \le 2 \end{cases}$$

The induced edge labeling are,

$$f^*(u_i u_{ij}) = \begin{cases} 0, 1 \le i \le \frac{m}{2}, 1 \le j \le 2 \\ 1, \frac{m+2}{2} \le i \le m, 1 \le j \le 2 \end{cases}$$

$$f^*(u_{i1} u_{i2}) = \begin{cases} 0, 1 \le i \le \frac{m}{2} \\ 1, \frac{m+2}{2} \le i \le m \end{cases}$$

$$f^*(u_{i2} u_{(i+1)1}) = \begin{cases} 0, 1 \le i \le \frac{m-2}{2} \\ 1, \frac{m}{2} \le i \le m-1 \end{cases}$$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(1) = e_f(0) + 1$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) – v_f(0) | \leq 1$

$| e_f(1) – e_f(0) | \leq 1$

Hence, $(C_3\otimes C_3)_m$ (m-even) is Mean Square Cordial Graph

For example, $(C_3\otimes C_3)_2$ is Mean Square Cordial Graph as shown in figure3.2.

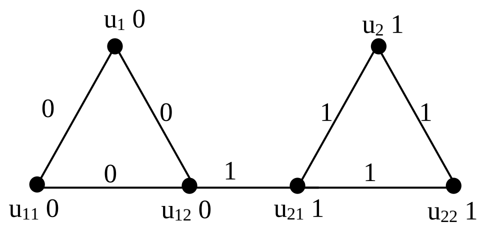

figure 3.2

Theorem: 3.3

$(C_3\otimes C_3)_m$ (m-odd) is Mean Square Cordial Graph.

Proof:

Let G be $(C_3\otimes C_3)_m$

Let $V(G) = \{ u_i , 1\leq i \leq m , u_{ij} : 1\leq i \leq m , 1\leq j \leq 2 \}$

Let $E(G) = \{ [(u_iu_{ij}) : 1\leq i \leq m , 1\leq j \leq 2] \cup [(u_{i1}u_{i2}) : 1\leq i \leq m] \cup [(u_{i2}u_{(i+1)1}) : 1\leq i \leq m-1]$

Define $f : V(G) \rightarrow \{0,1\}$

The vertex labeling are,

$$f(u_i) = \begin{cases} 0 , 1 \leq i \leq \frac{m+1}{2} \\ 1 , \frac{m+3}{2} \leq i \leq m \end{cases}$$

$$f(u_{i1}) = \begin{cases} 0 , 1 \leq i \leq \frac{m+1}{2} \\ 1 , \frac{m+3}{2} \leq i \leq m \end{cases}$$

$$f(u_{i2}) = \begin{cases} 0, 1 \le i \le \frac{m-1}{2} \\ 1, \frac{m+1}{2} \le i \le m \end{cases}$$

The induced edge labeling are,

$$f^*(u_i u_{i1}) = \begin{cases} 0, 1 \le i \le \frac{m+1}{2} \\ 1, \frac{m+3}{2} \le i \le m \end{cases}$$

$$f^*(u_i u_{i2}) = \begin{cases} 0, 1 \le i \le \frac{m-1}{2} \\ 1, \frac{m+1}{2} \le i \le m \end{cases}$$

$$f^*(u_{i1} u_{i2}) = \begin{cases} 0, 1 \le i \le \frac{m-1}{2} \\ 1, \frac{m+1}{2} \le i \le m \end{cases}$$

$$f^*(u_{i2} u_{(i+1)1}) = \begin{cases} 0, 1 \le i \le \frac{m-1}{2} \\ 1, \frac{m+1}{2} \le i \le m-1 \end{cases}$$

Here, $v_f(0) = v_f(1) + 1$ for all n and

$e_f(1) = e_f(0) + 1$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) - v_f(0) | \le 1$

$| e_f(1) - e_f(0) | \le 1$

Hence, $(C_3 \otimes C_3)_m$ (m-odd) is Mean Square Cordial Graph

For example, $(C_3 \otimes C_3)_3$ is Mean Square Cordial Graph as shown in figure 3.4.

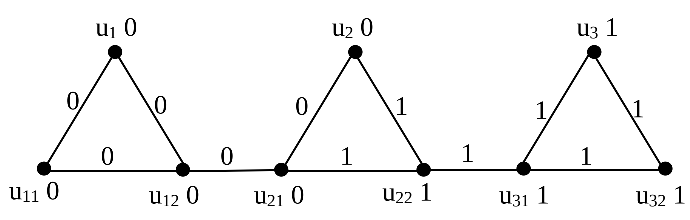

figure 3.4

Theorem: 3.5

Crown $C_n \odot K_1$ (n-odd) is Mean Square Cordial Graph.

Proof :

Let G be $C_n \odot K_1$

Let $V(G) = \{ u_i , v_i : 1 \leq i \leq n \}$

Let $E(G) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_n u_1)] \cup [(u_i v_i) : 1 \leq i \leq n] \}$

Define $f : V(G) \rightarrow \{0,1\}$

The vertex labeling are,

$f(u_i) = 0 , 1 \leq i \leq n$

$f(v_i) = 0 , 1 \leq i \leq n$

The induced edge labeling are,

$f^*(u_i u_{i+1}) =0 , \ 1 \leq i \leq n-1$

$f^*(u_n u_1) = 0$

$f^*(u_i v_i) = 1 \ , \ 1 \leq i \leq n$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1)$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) – v_f(0) | \leq 1$

$| e_f(1) – e_f(0) | \leq 1$

Hence, Crown $C_n \odot K_1$ (n-odd) is Mean Square Cordial Graph.

For example, Crown $C_3 \odot K_1$ is Mean Square Cordial Graph as shown in figure 3.6.

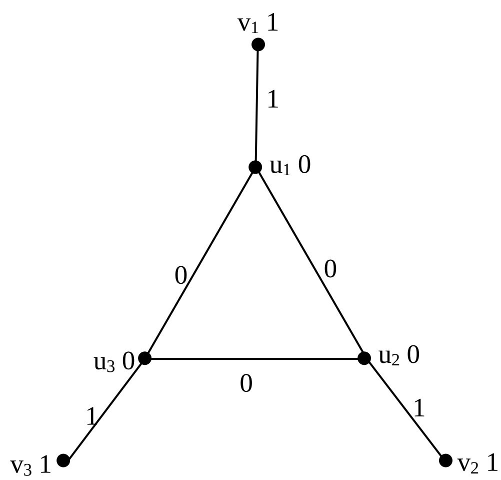

figure 3.6

Theorem: **3.7**

Double Triangular Snake $C_2(P_n)$ (n-odd) is Mean Square Cordial Graph.

Proof:

Let $V(C_2(P_n)) = \{ u_i : 1 \leq i \leq n , v_i , w_i : 1 \leq i \leq n-1 \}$

Let $E(C_2(P_n)) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i v_i) : 1 \leq i \leq n-1] \cup [(u_i w_i) : 1 \leq i \leq n-1] \cup$

$[(u_i v_{i-1}) : 2 \leq i \leq n] \cup [(u_i w_{i-1}) : 2 \leq i \leq n] \}$

Define $f : V(C_2(P_n)) \rightarrow \{0,1\}$

The vertex labeling are,

$$f(u_i) = \begin{cases} 0, 1 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n-1 \end{cases}$$

$$f(w_i) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n-1 \end{cases}$$

The induced edge labeling are,

$$f^*(u_i u_{i+1}) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n-1 \end{cases}$$

$$f^*(u_i v_i) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n-1 \end{cases}$$

$$f^*(u_i w_i) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n-1 \end{cases}$$

$$f^*(u_i v_{i-1}) = \begin{cases} 0, 2 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f^*(u_i w_{i-1}) = \begin{cases} 0, 2 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n \end{cases}$$

Here, $v_f(0) = v_f(1) + 1$ for all n and

$e_f(0) = e_f(1)$ for all n

Therefore, The Graph $C_2(P_n)$ satisfies the conditions

$| v_f(1) - v_f(0) | \leq 1$

$| e_f(1) – e_f(0) | \leq 1$

Hence , Double Triangular Snake $C_2(P_n)$ (n-odd) is Mean Square Cordial Graph.

For example, $C_2(P_5)$ is Mean Square Cordial Graph as shown in figure 3.8.

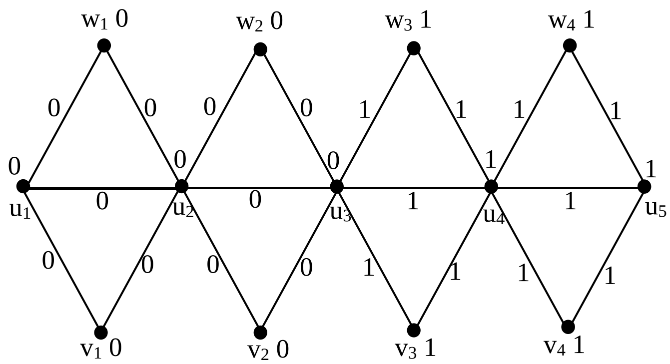

figure 3.8

Theorem: 3.9

Quadrilateral Snake Q(n) (n-odd) is Mean Square Cordial Graph.

Proof:

Let $V(Q(n)) = \{ u_i : 1\leq i \leq n ,\ v_i , w_i : 1\leq i \leq n-1 \}$

Let $E(Q(n)) = \{ [(u_iu_{i+1}) : 1\leq i \leq n-1] \cup [(u_iv_i) : 1\leq i \leq n-1] \cup [(v_iw_i) : 1\leq i \leq n-1]$

$\cup [(u_iw_{i-1}) : 2\leq i \leq n] \}$

Define $f : V(Q(n)) \rightarrow \{0,1\}$

The vertex labeling are,

$$f(u_i) = \begin{cases} 0 , 1 \leq i \leq \frac{n+1}{2} \\ 1 , \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} 0 , 1 \leq i \leq \frac{n-1}{2} \\ 1 , \frac{n+1}{2} \leq i \leq n-1 \end{cases}$$

$$f(w_i) = \begin{cases} 0 , 1 \leq i \leq \frac{n-1}{2} \\ 1 , \frac{n+1}{2} \leq i \leq n-1 \end{cases}$$

The induced edge labeling are,

$$f^*(u_iu_{i+1}) = \begin{cases} 0, 1 \le i \le \frac{n-1}{2} \\ 1, \frac{n+1}{2} \le i \le n-1 \end{cases}$$

$$f^*(u_iv_i) = \begin{cases} 0, 1 \le i \le \frac{n-1}{2} \\ 1, \frac{n+1}{2} \le i \le n-1 \end{cases}$$

$$f^*(v_iw_i) = \begin{cases} 0, 1 \le i \le \frac{n-1}{2} \\ 1, \frac{n+1}{2} \le i \le n-1 \end{cases}$$

$$f^*(u_iw_{i-1}) = \begin{cases} 0, 2 \le i \le \frac{n+1}{2} \\ 1, \frac{n+3}{2} \le i \le n \end{cases}$$

Here, $v_f(0) = v_f(1) + 1$ for all n and

$e_f(0) = e_f(1)$ for all n

Therefore, The Graph Q(n) satisfies the conditions

$| v_f(1) – v_f(0) | \le 1$

$| e_f(1) – e_f(0) | \le 1$

Hence, Quadrilateral Snake Q(n) (n-odd) is Mean Square Cordial Graph.

For example, Q(5) is Mean Square Cordial Graph as shown in figure 3.10.

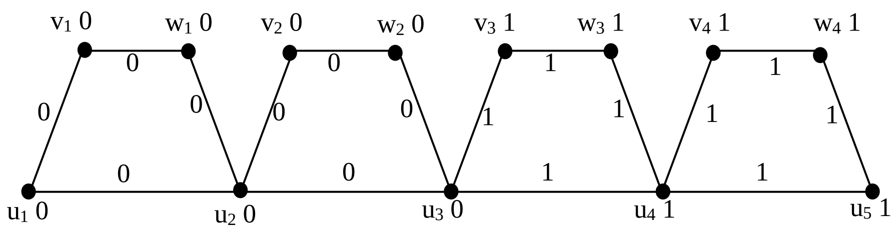

figure 3.10

IV. REFERENCES

1. Gallian. J.A,A *Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinotorics 6(2001)#DS6.

2. Harary,F.(1969), *Graph Theory*, Addision – Wesley Publishing Company Inc, USA.

3. A.Nellai Murugan (September 2011), *Studies in Graph theory- Some Labeling Problems in Graphs and Related topics*, Ph.D Thesis.

4. A.Nellai Murugan and V.Baby Suganya, Cordial labeling of path related splitted graphs, Indian Journal of Applied Research ISSN 2249 –555X,Vol.4, Issue 3, Mar. 2014, ISSN 2249 – 555X , PP 1-8. I .F . 2.1652

5. A.Nellai Murugan and M. Taj Nisha, A study on divisor cordial labelling of star attached paths and cycles, Indian Journal of Research ISSN 2250 –1991,Vol.3, Issue 3, Mar. 2014, PP 12-17. I .F . 1.6714.

6. A.Nellai Murugan and V.Brinda Devi, A study on path related divisor cordial graphs International Journal of Scientific Research, ISSN 2277–8179,Vol.3, Issue 4, April. 2014, PP 286 - 291. I .F . 1.8651.

7. A.Nellai Murugan and A Meenakshi Sundari, On Cordial Graphs International Journal of Scientific Research, ISSN 2277–8179,Vol.3, Issue 7 ,July. 2014, PP 54-55. I .F . 1.8651

8. A.Nellai Murugan and A Meenakshi Sundari, Results on Cycle related product cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968,Vol.I, Issue 5 ,July. 2014, PP 462-467.IF 0.611

9. A.Nellai Murugan and P.Iyadurai Selvaraj, Cycle and Armed Cup cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968,Vol.I, Issue 5 ,July. 2014,PP 478-485. IF 0.611

10. A.Nellai Murugan and G.Esther, Some Results on Mean Cordial Labelling , International Journal of Mathematics Trends and Technology ,ISSN 2231-5373,Volume 11, Number 2,July 2014,PP 97-101.

11. A.Nellai Murugan and A Meenakshi Sundari, Path related product cordial graphs, International Journal of Innovation in Science and Mathematics , ISSN 2347-9051,Vol 2., Issue 4 ,July 2014, PP 381-383

12. A.Nellai Murugan and P. Iyadurai Selvaraj, P*ath Related Cup Cordial graphs*, Indian Journal of Applied Research, ISSN 2249 –555X,Vol.4, Issue 8, August. 2014, PP 433-436.

13. A.Nellai Murugan , G.Devakiriba and S.Navaneethakrishnan, Star Attached Divisor cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968,Vol.I, Issue 6 ,August. 2014, PP 165-171.

14. A.Nellai Murugan and G. Devakiriba, Cycle Related Divisor Cordial Graphs, International Journal of Mathematics Trends and Technology , ISSN 2231-5373,Volume 12, Number 1,August 2014,PP 34-43.

15. A.Nellai Murugan and V.Baby Suganya, A study on cordial labeling of Splitting Graphs of star Attached C_3 and $(2k+1)C_3$ ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 142 -147. I.F 6.531

16. A.Nellai Murugan and V.Brinda Devi , A study on Star Related Divisor cordial Graphs ,ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 169-172. I.F 6.531

17. A.Nellai Murugan and M. Taj Nisha, A study on Divisor Cordial Labeling Star Attached Path Related Graphs, ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 173-178. I.F 6.531 .

18. .A .Nellai Murugan and V .Sripratha, Mean Square Cordial Labelling , International Journal of Innovative Research & Studies, ISSN 2319-9725 ,Volume 3, Issue 10Number 2 ,October 2014, PP 262-277.

19. A.Nellai Murugan and G. Esther , Path Related Mean Cordial Graphs , Journal of Global Research in Mathematical Archive , ISSN 2320 5822 , Volume 02, Number 3,March 2014,PP 74-86.

20. A. Nellai Murugan and A. Meenakshi Sundari , Some Special Product Cordial Graphs, Proceeding of the UGC Sponsored National Conference on Advances in Fuzzy Algebra , Fuzzy Topology and Fuzzay Graphs , Journal ENRICH , ISSN 2319-6394, January 2015, PP 129-141.

21. L. Pandiselvi ,S.Navaneethakrishan and A. Nellai Murugan ,Fibonacci divisor Cordial Cycle Related Graphs, Proceeding of the UGC Sponsored National Conference on Advances in Fuzzy Algebra ,

Fuzzy Topology and Fuzzay Graphs , Journal ENRICH , ISSN 2319-6394, January 2015, PP 142-150.