THE STUDY OF LONGITUDE DISPERSION PHENOMENON IN FLOW

PROBLEMS THROUGH LAPLACE TRANSFORM TECHNIQUE

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Abstract

The present paper discusses the phenomenon of longitudinal dispersion in the flow of two miscible fluids through porous media. An exact solution of partial differential equation arising in this phenomenon has been obtained by Laplace Transform method. A particular case of the former section by regarding flow velocity and dispersion coefficient as constants, and has been solved by method of Laplace transformation. The phenomenon of longitudinal dispersion is the process by which miscible fluids in laminar flow mix in the direction of flow has been discussed.

Key Words: Longitudinal, miscible, dispersion, similarity, Laplace

1. Introduction

The phenomenon of longitudinal dispersion in the flow of two miscible fluids through porous media. It is divided in two sections. First section deals with the solution by regarding cross-sectional flow velocity as time dependent, and dispersion coefficient as perturbation parameter that has been solved by using "similarity" transformation by singular perturbation technique. Second section is a particular case of the former section by regarding flow velocity and dispersion coefficient as constants, and has been solved by method of Laplace transformation.

The hydrodynamic dispersion is the macroscopic throughout the pores and various physical and chemical phenomenon that take place within the pores. This phenomenon simultaneously occurs due to molecular diffusion and convection. This phenomenon plays an important role in the sea water intrusion into at river mouths and in the underground recharge of waste water.

This problem has been discussed by several authors from different viewpoints, namely, Scheidegger [7],Greenkorn [3],Raval [6], Schwartz [8], Perrine [5],Hsiung Li [4], etc.

2. Explanation of the problem

The mathematical formulation of this specific problem of miscible fluid flow through porous media give rise to a partial differential equation. The particular case of this equation and solved by Laplace transform method.

The analysis of fluid flow, the flow is described with the average velocity of the fluid in a certain finite volume. Since fluid elements, in this volume are not actually moving with the same velocity, they tend to separate from one another as they flow. To account for this spreading of the fluid elements, it is necessary to regard the flow as capable of some dispersive action to be ascribed of the flow.

A fluid is considered to be a continuous material and hence in addition to the velocity of a fluid element, the molecules in this element have random motion. As a result of the random motion, molecules of a certain material in high concentration at one point will spread with time.so the velocity considered here is time dependent. The net molecular motion from a point of higher concentration to one of lower concentration is called molecular diffusion.

Fluid flows in nature are usually turbulent, but we have considered the porous medium through which the fluid flows, to be homogenous and for this reason, in the direction of flow, we assume laminar flow in which miscible fluid mix.

The geometrical dispersion is coupled with molecular diffusion and dispersion due to non-uniformity of the velocity across the cross-section of the passages. By considering the passage as randomly connected tubes, de Jong [1] and Safman [2] have shown that the dispersion in an isotropic medium can be desperation in the direction of seepage velocity.

3. Mathematical Formulation of the Problem

The equation of continuity for the mixture is given by

$$\frac{\partial \rho}{\partial t} + \nabla.\left(\rho \vec{V}\right) = 0 \tag{1}$$

Where ρ is the density for mixture and $\vec{V}$ is the pore velocity.

The equation of diffusion for a fluid flow through homogeneous porous medium with no addition or subtraction of the dispersing material, is given by

$$\frac{\partial C}{\partial t} + \nabla.\left(C\vec{V}\right) = \nabla.\left[\rho\, \bar{D}\, \nabla \left\{\frac{C}{\rho}\right\}\right] \qquad (2)$$

Where C is the concentration of the one fluid A in the other host fluid B (i.e. C is the mass of A per unit volume of the mixture), $\bar{D}$ is the tensor coefficient of dispersion with nine components D_{ij}.

In laminar flow through homogenous porous medium at constant temperature, ρ may be considered to be constant.

Then equation (2) gives

$$\nabla.\vec{V} = 0$$

And equation (2) become

$$\frac{\partial C}{\partial t} + \vec{V}.\nabla C = \nabla.[\,\bar{D}\ \nabla C] \qquad (3)$$

When the seepage velocity $\vec{V}$ is along the X- axis, the non-zero components are $D_{11} = D_L$ and $D_{22} = D_T$ (coefficient of transverse dispersion), and other D_{ij} are zero. Thus equation (3), in this case, become

$$\frac{\partial C}{\partial t} + u\,\frac{\partial C}{\partial x} = D_L \frac{\partial^2 C}{\partial x^2} \qquad (4)$$

Where u is the component of velocity along X-axis which is the dependent and D_L is the longitudinal dispersion coefficient.

4. Solution of the Problem

We consider a particular case of (4) by using Laplace Transform method. We assume, in this equation the flow velocity u and the diffusivity coefficient D_L to be constants.

An appropriate boundary condition may be written as :

$$C(0,t) = \gamma_1 \quad and \qquad C(\infty,t) = 0 \ \ for \ \ t > 0 \qquad (5)$$

Let $C(X,S) = L\{C(x,t)\}$, be the Laplace transform with respect to t. Applying Laplace Transform on both sides of equation (1) to obtain an ordinary differential equation with constant coefficients

$$D_L \frac{d^2C}{dx^2} - u\frac{dC}{dX} - sC = 0 \tag{6}$$

The solution of this equation is

$$C(X,S) = A(s)\exp\left[\frac{uX}{2D_L} - \frac{X}{\sqrt{D_L}}\sqrt{\frac{u^2}{4D_L} + S}\right] + \tag{7}$$

$$B(s)\exp\left[\frac{uX}{2D_L} - \frac{X}{\sqrt{D_L}}\sqrt{\frac{u^2}{4D_L} + S}\right]$$

The condition (5) becomes

$$C(0,t) = \frac{\gamma_1}{s} \quad and \quad C(\infty,t) = 0 \tag{8}$$

Using conditions (8) in equation (7) we can determine the values of arbitrary constants as follows:

$$\text{A(s)} = \frac{\gamma_1}{s} \quad \text{and} \quad B(s) = 0$$

Thus equation (7) becomes

$$C(X,S) = \frac{\gamma_1}{s}\exp\left[\frac{uX}{2D_L} - \frac{X}{\sqrt{D_L}}\sqrt{\frac{u^2}{4D_L} + S}\right] \tag{9}$$

Now taking inverse Laplace transformation of equation (9) and using the properties of inverse Laplace transformation we get

$$\frac{C(X,t)}{\gamma_1} = \frac{1}{2}\,erfc\left[\frac{X-ut}{2\sqrt{D_L t}}\right] + \frac{1}{2}\exp\left\{\frac{uX}{D_L}\right\} erfc\left[\frac{X+ut}{2\sqrt{D_L t}}\right] \tag{10}$$

5. Conclusion

Equation (10) is the Laplace transform solution of miscible fluid flow through porous media under certain conditions and assumptions. The solution contains special functions such as an exponential and complementary error function. The first term on the right side indicates the spreading of moving miscible fluid at $X = ut$. The last tern is needed to satisfy the condition $C = \gamma_1$ at $X = 0$.The numerical values of concentration from the solution by using available tables for special functions in standard literature [2].Due to our practical interest in an analytical study, we have not interpreted it numerically and graphically.

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