Exact Solutions For Fractional Differential Equations Arising in the Teaching of Mathematical Physical Equations By The Improved (G'/G) Method

Bin Zheng,[1], Qinghua Feng[1,*]
[1]Shandong University of Technology
School of Mathematics and Statistics
Zhangzhou Road 12, Zibo, 255049, China
fqhua@sina.com

Abstract: In this paper, we are concerned with seeking exact solutions for space-time fractional differential equations arising in the teaching of the college course mathematical physical equations. The improved (G'/G) method is extended to seek exact solutions for fractional differential equations in the sense of the conformable fractional derivative. Based on a fractional complex transformation, a certain fractional differential equation can be converted into another ordinary differential equation of integer order, and then can be solved subsequently based on the homogeneous balance principle. As for applications of this method, we apply it to solve the (2+1)-dimensional space-time fractional Nizhnik-Novikov-Veselov System, and as a result, construct some new exact solutions for it.

Key–Words: Improved (G'/G) method; Fractional differential equations; Exact solutions; Fractional complex transformation; (2+1)-dimensional space-time fractional Nizhnik-Novikov-Veselov System.

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1 Introduction

Fractional differential equations play important roles in the teaching of mathematical physical equations, and have attracted much attention in a variety of applied sciences including physics, dynamical systems, control systems, engineering and so on. Furthermore, they are employed in social sciences such as food supplement, climate, and economics. Recently there have been an increasing attention to fractional differential equations due to their frequent appearance in wide applications in various fields. Many practical problems lead to the necessity of seeking exact solutions and numerical solutions for fractional differential equations. Many powerful and efficient methods have been proposed so far, such as the fractional Jacobi elliptic equation method [1], the fractional sub-equation method [2-4], the variational iterative method [5-6], the Adomian decomposition method [7], the homotopy perturbation method [8-10] and so on. Based on these methods, a variety of fractional differential equations have been investigated.

In this paper, we extend the known improved (G'/G) method [11] to seek exact solutions for fractional differential equations. The fractional differential equations concerned are defined in the sense of the conformable fractional derivative [12]. Based on a fractional complex transformation, a certain fractional differential equation can be converted into another differential equation of integer order, and then can be solved subsequently based on the homogeneous balance principle. For demonstrating the validity of the two methods, we apply it to construct exact solutions for (2+1)-dimensional space-time fractional Nizhnik-Novikov-Veselov System.

The definition and some important properties of the conformable fractional derivative [12] are listed as follows.

$$D^{\alpha}f(t)=\lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon}. \tag{1}$$
$$D_t^{\alpha}(t^{\gamma})=\gamma t^{\gamma-\alpha}. \tag{2}$$
$$D_t^{\alpha}(f(t)g(t))=g(t)D_t^{\alpha}f(t)+f(t)D_t^{\alpha}g(t). \tag{3}$$

$$D_t^\alpha f[g(t)] = f'_g[g(t)]D_t^\alpha g(t). \tag{4}$$
$$D_t^\alpha(\frac{f}{g})(t) = \frac{g(t)D^\alpha f(t) - f(t)D^\alpha g(t)}{g^2(t)}. \tag{5}$$

The rest of this paper is organized as follows. In Section 2, we give the description of the improved (G'/G) method for seeking exact solutions of fractional differential equations. In Section 3, we apply the improved (G'/G) method to construct exact solutions for the (2+1)-dimensional space-time fractional Nizhnik-Novikov-Veselov System. Some conclusions are presented at the end of the paper.

2 Description of the improved (G'/G) method for fractional differential equations

In this section we give the description of the improved (G'/G) method for seeking exact solutions of fractional differential equations.

Suppose that a fractional partial differential equation, say in the independent variables $t, x_1, x_2, ..., x_n$, is given by

$$P(u_1, ...u_k,\ D_t^\alpha u_1, ..., D_t^\alpha u_k, D_{x_1}^\beta u_1, ..., D_{x_1}^\beta u_k, ..., D_{x_n}^\gamma u_1, ..., D_{x_n}^\gamma u_k, ...) = 0, \tag{6}$$

where $u_i = u_i(t, x_1, x_2, ..., x_n),\ i = 1, ..., k$ are unknown functions, P is a polynomial in u_i and their various partial derivatives including fractional derivatives.

Step 1. Execute certain variable transformation

$$u_i(t, x_1, x_2, ..., x_n) = U_i(\xi), \quad \xi = \xi(t, x_1, x_2, ..., x_n), \tag{7}$$

such that Eq. (6) can be turned into the following ordinary differential equation of integer order with respect to the variable ξ:

$$\widetilde{P}(U_1, ..., U_k,\ U_1', ..., U_k', U_1'', ..., U_k'',\ ...) = 0. \tag{8}$$

Step 2. Suppose that the solution of (8) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$U_j(\xi) = \sum_{i=0}^{m_j} a_{j,i}(\frac{G'}{G})^i,\ j = 1, 2, ..., k, \tag{9}$$

where $a_{j,i},\ i = 0, 1, ..., m_j,\ j = 1, 2, ..., k$ are constants to be determined later, $a_{j,m} \neq 0$, the positive integer m_j can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (8), $G = G(\xi)$ satisfies the second order ODE in the form

$$AGG'' - BGG' - C(G')^2 - EG^2 = 0, \tag{10}$$

where $A,\ B,\ C,\ E$ are real parameters.

Denote

$$\Delta_1 = B^2 + 4E(A - C),\ \Delta_2 = E(A - C). \tag{11}$$

By the general solutions of Eq. (10) [11] we have the following expressions for $(\frac{G'(\xi)}{G(\xi)})$:

When $B \neq 0,\ \Delta_1 > 0$:

$$(\frac{G'(\xi)}{G(\xi)}) = \frac{B}{2(A-C)} + \frac{\sqrt{\Delta_1}}{2(A-C)}[\frac{C_1 \sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)} + C_2 \cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}{C_1 \cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)} + C_2 \sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}], \tag{12}$$

where $C_1,\ C_2$ are arbitrary constants.

When $B \neq 0,\ \Delta_1 < 0$:

$$(\frac{G'(\xi)}{G(\xi)})=\frac{B}{2(A-C)}+\frac{\sqrt{-\Delta_1}}{2(A-C)}[\frac{-C_1\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}{C_1\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}], \tag{13}$$

where $C_1,\ C_2$ are arbitrary constants.

When $B\neq 0,\ \Delta_1=0$:

$$(\frac{G'(\xi)}{G(\xi)})=\frac{B}{2(A-C)}+\frac{C_2}{C_1+C_2\xi}, \tag{14}$$

where $C_1,\ C_2$ are arbitrary constants.

When $B=0,\ \Delta_2>0$:

$$(\frac{G'(\xi)}{G(\xi)})=\frac{\sqrt{\Delta_2}}{(A-C)}[\frac{C_1\sinh\frac{\sqrt{\Delta_2}\xi}{(A-C)}+C_2\cosh\frac{\sqrt{\Delta_2}\xi}{(A-C)}}{C_1\cosh\frac{\sqrt{\Delta_2}\xi}{(A-C)}+C_2\sinh\frac{\sqrt{\Delta_2}\xi}{(A-C)}}], \tag{15}$$

where $C_1,\ C_2$ are arbitrary constants.

When $B=0,\ \Delta_2<0$:

$$(\frac{G'(\xi)}{G(\xi)})=\frac{\sqrt{-\Delta_2}}{(A-C)}[\frac{-C_1\sin\frac{\sqrt{-\Delta_2}\xi}{(A-C)}+C_2\cos\frac{\sqrt{-\Delta_2}\xi}{(A-C)}}{C_1\cos\frac{\sqrt{-\Delta_2}\xi}{(A-C)}+C_2\sin\frac{\sqrt{-\Delta_2}\xi}{(A-C)}}], \tag{16}$$

where $C_1,\ C_2$ are arbitrary constants.

Step 3. Substituting (9) into (8) and using (10), collecting all terms with the same order of $(\frac{G'}{G})$ together, the left-hand side of (8) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $a_{j,i},\ i=0,1,...,m,\ j=1,2,...,k$.

Step 4. Solving the equations system in Step 3, and using (12)-(16), we can construct a variety of exact solutions for Eq. (6).

3 Application of the improved (G'/G) method to the (2+1)-dimensional space-time fractional Nizhnik-Novikov-Veselov System

In this section, we apply the improved (G'/G) method described in Section 2 to seek exact solutions for the (2+1)-dimensional space-time fractional Nizhnik-Novikov-Veselov System [13]

$$\begin{cases} D_t^{\alpha}u+aD_x^{3\beta}u+bD_y^{3\gamma}u+cD_x^{\beta}u+dD_y^{\gamma}u \\ \qquad =3aD_x^{\beta}(uv)+3bD_y^{\gamma}(uw), \\ D_x^{\beta}u=D_y^{\gamma}v, \\ D_y^{\gamma}u=D_x^{\beta}w, \end{cases} ,0<\alpha,\ \beta,\ \gamma\leq 1, \tag{17}$$

which is a fractional extension of the following (2+1)-dimensional Nizhnik-Novikov-Veselov system [14]:

$$\begin{cases} u_t+au_{xxx}+bu_{yyy}+cu_x+du_y=3a(uv)_x+3b(uw)_y, \\ u_x=v_y, \\ u_y=w_x. \end{cases} \tag{18}$$

In order to apply the improved (G'/G) method, we suppose $u(x,y,t)=U(\xi)$, $v(x,y,t)=V(\xi)$, $w(x,y,t)=W(\xi)$, where $\xi=\frac{m}{\alpha}t^{\alpha}+\frac{k}{\beta}x^{\beta}+\frac{l}{\gamma}y^{\gamma}+\xi_0$, $m,\ k,\ l,\ \xi_0$ are all constants with $k,\ l,\ m\neq 0$. By use of (2) and (4), we obtain

$$\begin{cases} D_t^{\alpha}u=D_t^{\alpha}U(\xi)=U'(\xi)D_t^{\alpha}\xi=mU'(\xi),\\ D_x^{\beta}u=D_x^{\beta}U(\xi)=U'(\xi)D_x^{\beta}\xi=kU'(\xi),\\ D_y^{\gamma}u=D_y^{\gamma}U(\xi)=U'(\xi)D_y^{\gamma}\xi=lU'(\xi),\end{cases}$$

and then Eqs. (17) can be turned into the following forms

$$\begin{cases} mU'+ak^3U'''+bl^3U'''+ckU'+dlU'=3ak(UV)'+3bl(UW)',\\ kU'=lV',\\ lU'=kW'.\end{cases}\tag{19}$$

Integrating (19) once, we have

$$\begin{cases} mU+ak^3U''+bl^3U''+ckU+dlU=3akUV+3blUW+g_1,\\ kU=lV+g_2,\\ lU=kW+g_3,\end{cases}\tag{20}$$

where g_1,g_2,g_3 are the integration constants.

Suppose that the solutions of (20) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$U(\xi)=\sum_{i=0}^{m_1}a_i(\frac{G'}{G})^i,\ V(\xi)=\sum_{i=0}^{m_2}b_i(\frac{G'}{G})^i,\ W(\xi)=\sum_{i=0}^{m_3}c_i(\frac{G'}{G})^i,\tag{21}$$

where a_i,b_i,c_i are constants, and $G=G(\xi)$ satisfies Eq. (10). Balancing the order of u'' and uv, the order of u and v, the order of u and w in (20), we can obtain $m_1=m_2=m_3=2$. So we have

$$U(\xi)=a_0+a_1(\frac{G'}{G})^1+a_2(\frac{G'}{G})^2,\ V(\xi)=b_0+b_1(\frac{G'}{G})^1+b_2(\frac{G'}{G})^2,\ W(\xi)=c_0+c_1(\frac{G'}{G})^1+c_2(\frac{G'}{G})^2.\tag{22}$$

Substituting (22) into (20), using Eq. (10) and collecting all the terms with the same power of $(\frac{G'}{G})$ together, equating each coefficient to zero, yields a set of algebraic equations. Solving these equations, yields:

Case 1:

$$a_2=\frac{2lk(-C+A)^2}{A^2},\ a_1=-\frac{2klB(-C+A)}{A^2},\ a_0=-\frac{2klE(-C+A)}{A^2},\ g_1=g_2=g_3=0$$

$$b_2=\frac{2k^2(-C+A)^2}{A^2},\ b_1=-\frac{2k^2B(-C+A)}{A^2},\ b_0=-\frac{2k^2E(-C+A)}{A^2},$$

$$c_2=\frac{2l^2(-C+A)^2}{A^2},\ c_1=-\frac{2l^2B(-C+A)}{A^2},\ c_0=-\frac{2l^2E(-C+A)}{A^2},$$

$$m=-\frac{dlA^2+ckA^2+4bl^3AE+4ak^3AE+bl^3B^2-4ak^3CE+ak^3B^2-4bl^3CE}{A^2}.$$

Case 2:

$$a_2=\frac{2lk(A^2-2CA+C^2)}{A^2},\ a_1=-\frac{2klB(-C+A)}{A^2},\ a_0=-\frac{lk(2AE-2CE-B^2)}{3A^2},$$

$$b_2=\frac{2k^2(A^2-2CA+C^2)}{A^2},\ b_1=-\frac{2k^2B(-C+A)}{A^2},\ b_0=-\frac{k^2(2AE-2CE-B^2)}{3A^2},$$

$$c_2=\frac{2l^2(A^2-2CA+C^2)}{A^2},\ c_1=-\frac{2l^2B(-C+A)}{A^2},\ c_0=-\frac{l^2(2AE-2CE-B^2)}{3A^2},$$
$$m=-\frac{dlA^2+ckA^2-4bl^3AE-4ak^3AE-bl^3B^2+4ak^3CE-ak^3B^2+4bl^3CE}{A^2},$$
$$g_1=g_2=g_3=0.$$

Substituting the results above into Eqs. (22) and combining with (12)-(16) we can obtain corresponding exact solutions for Eqs. (18).

For example, from Case 1 and the combination with (12)-(14) we get the following three families of exact solutions, where $C_1,\ C_2$ are arbitrary constants, and

$$\xi=[-\frac{dlA^2+ckA^2+4bl^3AE+4ak^3AE+bl^3B^2-4ak^3CE+ak^3B^2-4bl^3CE}{A^2}]$$
$$\times\frac{t^\alpha}{\alpha}+\frac{k}{\beta}x^\beta+\frac{l}{\gamma}y^\gamma+\xi_0.$$

Family 1: when $B\neq 0,\ \Delta_1>0$:

$$u_1(x,y,t)=-\frac{2klE(-C+A)}{A^2}-\frac{2klB(-C+A)}{A^2}\times$$
$$\{\frac{B}{2(A-C)}+\frac{\sqrt{\Delta_1}}{2(A-C)}[\frac{C_1\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}{C_1\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}]\}^1$$
$$+\frac{2lk(-C+A)^2}{A^2}\{\frac{B}{2(A-C)}+\frac{\sqrt{\Delta_1}}{2(A-C)}[\frac{C_1\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}{C_1\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}]\}^2,$$

$$v_1(x,y,t)=-\frac{2k^2E(-C+A)}{A^2}-\frac{2k^2B(-C+A)}{A^2}\times$$
$$\{\frac{B}{2(A-C)}+\frac{\sqrt{\Delta_1}}{2(A-C)}[\frac{C_1\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}{C_1\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}]\}^1$$
$$+\frac{2k^2(-C+A)^2}{A^2}\{\frac{B}{2(A-C)}+\frac{\sqrt{\Delta_1}}{2(A-C)}[\frac{C_1\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}{C_1\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}]\}^2,$$

$$w_1(x,y,t)=-\frac{2l^2E(-C+A)}{A^2}-\frac{2l^2B(-C+A)}{A^2}\times$$
$$\{\frac{B}{2(A-C)}+\frac{\sqrt{\Delta_1}}{2(A-C)}[\frac{C_1\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}{C_1\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}]\}^1$$
$$+\frac{2l^2(-C+A)^2}{A^2}\{\frac{B}{2(A-C)}+\frac{\sqrt{\Delta_1}}{2(A-C)}[\frac{C_1\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}{C_1\cosh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}+C_2\sinh\frac{\sqrt{\Delta_1}\xi}{2(A-C)}}]\}^2.$$

Family 2: when $B\neq 0,\ \Delta_1<0$:

$$u_2(x,y,t)=-\frac{2klE(-C+A)}{A^2}-\frac{2klB(-C+A)}{A^2}\times$$

$$\{\frac{B}{2(A-C)}+\frac{\sqrt{-\Delta_1}}{2(A-C)}[\frac{-C_1\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}{C_1\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}]\}^1$$

$$+\frac{2lk(-C+A)^2}{A^2}\{\frac{B}{2(A-C)}+\frac{\sqrt{-\Delta_1}}{2(A-C)}[\frac{-C_1\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}{C_1\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}]\}^2,$$

$$v_2(x,y,t)=-\frac{2k^2E(-C+A)}{A^2}-\frac{2k^2B(-C+A)}{A^2}\times$$

$$\{\frac{B}{2(A-C)}+\frac{\sqrt{-\Delta_1}}{2(A-C)}[\frac{-C_1\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}{C_1\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}]\}^1$$

$$+\frac{2k^2(-C+A)^2}{A^2}\{\frac{B}{2(A-C)}+\frac{\sqrt{-\Delta_1}}{2(A-C)}[\frac{-C_1\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}{C_1\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}]\}^2,$$

$$w_2(x,y,t)=-\frac{2l^2E(-C+A)}{A^2}-\frac{2l^2B(-C+A)}{A^2}\times$$

$$\{\frac{B}{2(A-C)}+\frac{\sqrt{-\Delta_1}}{2(A-C)}[\frac{-C_1\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}{C_1\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}]\}^1$$

$$+\frac{2l^2(-C+A)^2}{A^2}\{\frac{B}{2(A-C)}+\frac{\sqrt{-\Delta_1}}{2(A-C)}[\frac{-C_1\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}{C_1\cos\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}+C_2\sin\frac{\sqrt{-\Delta_1}\xi}{2(A-C)}}]\}^2.$$

Family 3: when $B\neq 0,\ \Delta_1=0$:

$$u_3(x,y,t)=-\frac{2klE(-C+A)}{A^2}-\frac{2klB(-C+A)}{A^2}\{\frac{B}{2(A-C)}+\frac{C_2}{C_1+C_2\xi}\}^1$$

$$+\frac{2lk(-C+A)^2}{A^2}\{\frac{B}{2(A-C)}+\frac{C_2}{C_1+C_2\xi}\}^2,$$

$$v_3(x,y,t)=-\frac{2k^2E(-C+A)}{A^2}-\frac{2k^2B(-C+A)}{A^2}\{\frac{B}{2(A-C)}+\frac{C_2}{C_1+C_2\xi}\}^1\times$$

$$+\frac{2k^2(-C+A)^2}{A^2}\{\frac{B}{2(A-C)}+\frac{C_2}{C_1+C_2\xi}\}^2,$$

$$w_3(x,y,t)=-\frac{2l^2E(-C+A)}{A^2}-\frac{2l^2B(-C+A)}{A^2}\{\frac{B}{2(A-C)}+\frac{C_2}{C_1+C_2\xi}\}^1\times$$

$$+\frac{2l^2(-C+A)^2}{A^2}\{\frac{B}{2(A-C)}+\frac{C_2}{C_1+C_2\xi}\}^2.$$

From Case 2 and the combination with (12)-(16) we can obtain corresponding exact solutions similar above, which are omitted here.

Remark. If we put $A=1,\ B=-\lambda,\ C=0,\ E=-\mu$, then the solutions denoted in Families 1-3 above reduce to the results established in [13, Eqs. (24)-(26)]. Furthermore, if we put $A=1,\ B=-\lambda,\ C=0,\ E=-\mu,\ \alpha=1,\ k=l=1$, then the solutions in Families 1-3 reduce to the results established in [14] despite the slight difference of the forms of constants. So the solutions obtained here are of more general forms than those in [13,14].

4 Conclusions

The improved (G'/G) method have been extended to seek exact solutions for fractional differential equations arising in the teaching of mathematical physical equations. The most important point lies in the fractional complex transformation put on fractional differential equations. Based on the fractional complex transformation, certain fractional differential equation can be converted into another ordinary differential equation of integer order, and then can be solved subsequently based on the homogeneous balance principle. As for applications, we apply this method to solve the (2+1)-dimensional space-time fractional Nizhnik-Novikov-Veselov System. With the aid of the mathematical software, some new exact solutions have been successfully found. Finally, we note that this method can be easily applied to solve many other fractional differential equations.

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